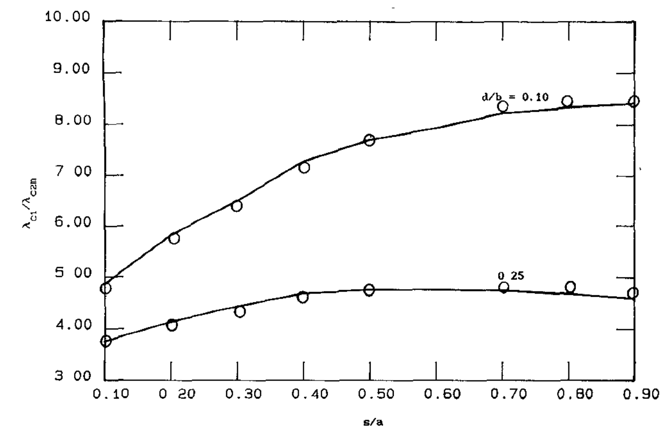

Fig. 6. Comparison of third mode from TLM analysis and second mode from analysis in [4] for DTSG ($b/a = 0.50$, $w/a = 0.10$, $t/b = 0.50$) O Mazumder and Saha; —— TLM.

IV. Conclusions

The bandwidth characteristics of rectangular waveguide with both single and double T-shaped septa have been investigated. The numerical analysis was performed using the well-established TLM method. Results for the cutoff waveguide of the dominant mode have been calculated for various dimensions of the structure and have been found to be in very good agreement with previously calculated results. Previously, there was disagreement in the correct results for the bandwidth characteristics of the guides. Using the TLM simulation, we have calculated the bandwidth for both the STSG and the DTSG. Our results are shown to agree with those originally given by Mazumder and Saha [2], [3]. It has also been observed that the results for bandwidth calculated by Zhang and Joines [4] appear to actually be the separation between the first and third TE modes.

Considering the agreement of the results obtained using the Ritz–Galerkin technique and the TLM method, it is felt that a great deal of confidence may be placed in the results of this and previous investigations.

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On the Bandwidth of T-Septum Waveguide

B. E. PAUPLIS AND D. C. POWER

***Abstract*—There is some degree of confusion in the recent literature concerning the bandwidth properties of T-septum waveguide. Two analyses, using the same Rayleigh–Ritz–Galerkin technique, disagree significantly. We present another analysis of the T-septum guide, using an alternative formulation of the method, which supports the original analysis of Mazumder and Saha. A possible source of the disagreement is found and illustrated. The history of the T-septum waveguide is discussed and several additional references are provided.**

I. Introduction

The history of the T-septum waveguide is long and interesting. This type of waveguide seems to have appeared first in the literature in 1968, when Elliott [1] proposed and analyzed it as an example of a waveguide having two modes which can be made to propagate at a common phase velocity. The proposed application was to slotted guide antennas having variable polarization. Although Elliott was particularly interested in a configuration with a single T septum attached to the sidewall of rectangular waveguide, his analysis did not depend on whether the septum was attached to the narrow or to the broad wall. An additional analysis of the configuration was presented by Silvester [2] as an example of the use of a finite element analysis program for general waveguide cross sections. The T-septum guide also was analyzed by Alexopoulos and Armstrong [3] and by Lyon [4]. The primary interest of all of these authors in the structure was as a two-mode waveguide with equal mode velocities, but the wideband possibilities of the configuration were understood [1], [4].

Recently, Mazumder and Saha [5], [6] generalized the waveguide cross section to include double T-septum guide. They reported the analysis of the cutoff frequencies and bandwidths of various geometries, with the principal motivation being the optimization of bandwidth. They discovered that the waveguide could be designed to have slightly broader bandwidths than standard single- or dual-ridged guide. The most recent work concerning T-septum guide was that of Zhang and Joines [7], [8], who reported that [5] and [6] had incorrectly identified the second mode eigenvalue and therefore the waveguide was significantly broader in bandwidth than had been believed. The reported discrepancy is significant; the differences between the analyses approach 75 percent in some cases. Both groups used the Rayleigh–Ritz–Galerkin procedure originally described by Montgomery [9] and subdivided the waveguide cross section as shown in Fig. 1(a).

This paper reports the results of our analysis of the T-septum waveguide. We also use the Rayleigh–Ritz–Galerkin procedure, but we subdivide the waveguide cross section as shown in Fig. 1(b). We find that our results agree very closely with the original analyses in [1]–[6] and with the new results of German and Riggs [10], but disagree significantly with the results in [7]. We indicate what we believe to be the error in [7] by reproducing their results using our analysis. We further show that the usual practice of naming the modes of reentrant waveguide, such as the T-septum and ridged guide, with the standard nomenclature of rectangular waveguide (TE_{mn}) is misleading.

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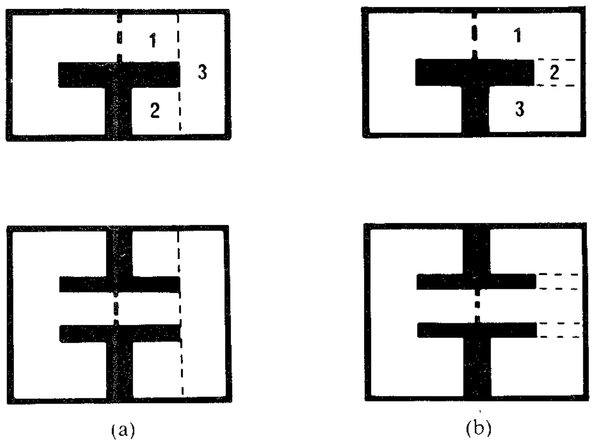

Fig. 1. (a) Transverse cross section for single T-septum waveguide (top) and double T-septum waveguide (bottom). The heavy dashed lines indicate the symmetry wall on which transverse fields obey either magnetic (for symmetric modes) or electric (for antisymmetric) boundary conditions. The light dashed lines and the numbers indicate the subdivision scheme used by Mazumder and Saha and by Zhang and Joines. (b) Same as (a) except the light dashed lines indicate the subdivision scheme used in the present analysis.

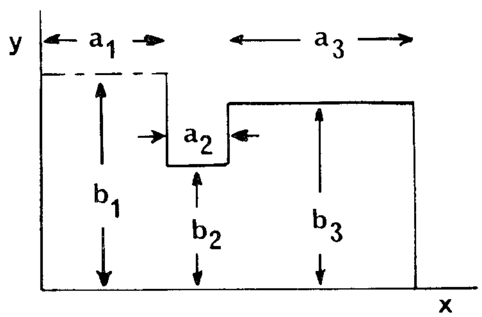

Fig. 2. Geometry and parameter labels used in the analysis. The dashed line indicates the symmetry wall at the center of the waveguide We have rotated the coordinate system 90° from the previous authors in order to agree with some of our previous work on ridged guide

II. Analysis

The Rayleigh–Ritz–Galerkin method used in our analysis is a standard procedure (see [9]), so only an outline of the derivation of the eigenvalue equation is presented below. The derivation for the double T-septum guide (DTSG) is a simple generalization of the single T-septum guide equations, so only the STSG case is treated. Since the waveguide is homogeneously filled, the modes divide naturally into transverse electric (TE) and transverse magnetic (TM) but we find that at least four TE modes have cutoff frequencies below the first TM mode; therefore only the TE case is considered. The geometry is illustrated in Fig. 1(b) and the precise parameter definitions are shown in Fig. 2. Note that the notation we use is rotated 90° from that used in the previous analyses.

Matching tangential electric and magnetic fields at the region boundaries results in an integral eigenvalue problem which is then solved by Galerkin's method. The set of coupled homogeneous equations which results is

$$\sum_{j=0}^{J} C_{1j} \sum_{m=0}^{M} V_{1m} X_{1jm} X_{1qm} + C_{1q} T_q - C_{2q} S_q = 0 \qquad (1)$$

$$-C_{1q} S_q + C_{2q} T_q + \sum_{j=0}^{J} C_{2j} \sum_{n=0}^{N} V_{3n} X_{2jn} X_{2qn} = 0 \qquad (2)$$

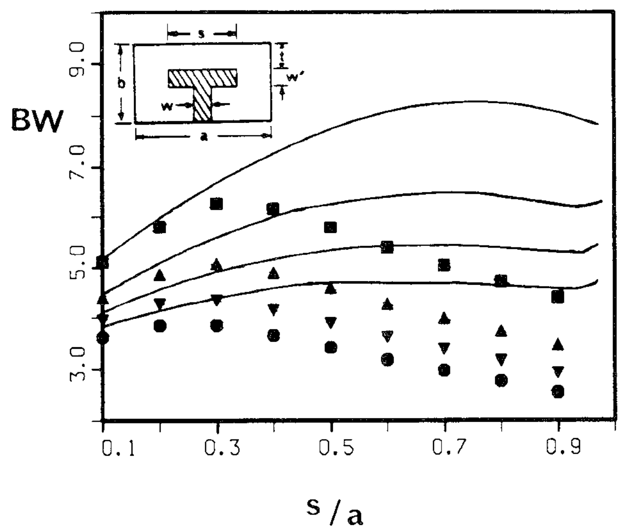

Fig 3. Comparison of our results with those of Zhang and Joines for the bandwidth of single T-septum guide The solid curves indicate their results for $t/b = 0.10$, 0.15, 0 20, and 0 25 from top to bottom The symbols indicate our results for $t/b = 0.10$ (squares), $t/b = 0\ 15$ (triangles), $t/b = 0.20$ (inverted triangles), and $t/b = 0\ 25$ (circles). Values for the other parameters are $b/a = 0.45$, $w'/b = 0\ 05$, and $w/a = 0.10$.

where

$$V_{ij} = \left[k_{xij} \epsilon_j b_i \tan(k_{xij} a_i) \right]^{-1}$$

$$S_m = \epsilon_m b_2 / \left[k_{x2m} \sin(k_{x2m} a_2) \right]$$

$$T_m = \epsilon_m b_2 / \left[k_{x2m} \tan(k_{x2m} a_2) \right]$$

$$\epsilon_m = \begin{cases} 1 & \text{for } m = 0 \\ 1/2 & \text{for } m \neq 0 \end{cases}$$

$$X_{1im} = \int_0^{b_2} \cos(i\pi y/b_2) \begin{Bmatrix} \cos[(m+1/2)\pi y/b_1] \\ \cos[m\pi y/b_1] \end{Bmatrix} dy$$

and

$$X_{2jn} = \int_0^{b_2} \cos(j\pi y/b_2) \cos(n\pi y/b_3)\, dy.$$

The upper term in curly brackets in the definition of X_{1im} is for the magnetic symmetry wall case and the lower term is for the electric symmetry wall case. It is of interest to observe that with this scheme of subdividing the waveguide cross section, the electric wall case reduces to a generalized form of single-ridged waveguide. A similar reduction occurs in the DTSG electric wall case to a geometry which has been studied previously [11]–[13]. The cutoff frequencies of the various modes are obtained by setting to zero the determinant of the coefficient matrix defined by (1) and (2).

In order to preclude the possibility of relative convergence we use the procedure described in [14]. The idea is described most easily by assuming that J is the number of expansion modes used to expand the Hertz potential in region 2. The number of modes in regions 1 and 3 are M and N, respectively. The value of J is chosen (typically $J = 8$ or 10) and the values of M and N are determined by the relationships

$$M = \text{integer part of } \left[(b_1/b_2) * J \right] \qquad (3)$$

$$N = \text{integer part of } \left[(b_3/b_2) * J \right]. \qquad (4)$$

The analysis of the DTSG proceeds in precisely similar fashion except that there are four coupled equations in the homogeneous system.

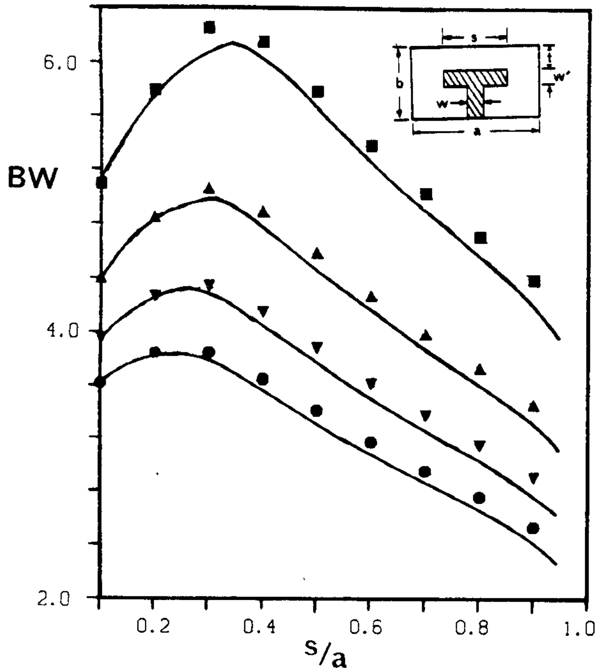

Fig. 4. Comparison of our results with those of Mazumder and Saha for the bandwidth of single T-septum guide. The solid curves represent their results for $t/b = 0.10$, 0.15, 0.20, and 0.25 from top to bottom. The symbols indicate our results for $t/b = 0.10$ (squares), $t/b = 0.15$ (triangles), $t/b = 0.20$ (inverted triangles), and $t/b = 0.25$ (circles). Values for the other parameters are $b/a = 0.45$, $w'/b = 0.05$, and $w/a = 0.10$.

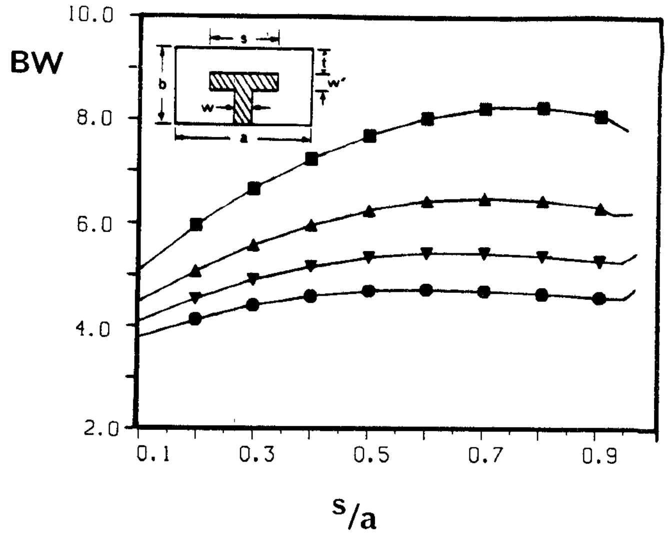

Fig. 5. Comparison of Zhang and Joines's results for $t/b = 0.10$, 0.15, 0.20, and 0.25 (from top to bottom) with our results. The symbols indicate the difference between the dominant mode and next higher mode associated with a magnetic wall. Both modes are symmetric and represent the first and third propagating modes of the structure.

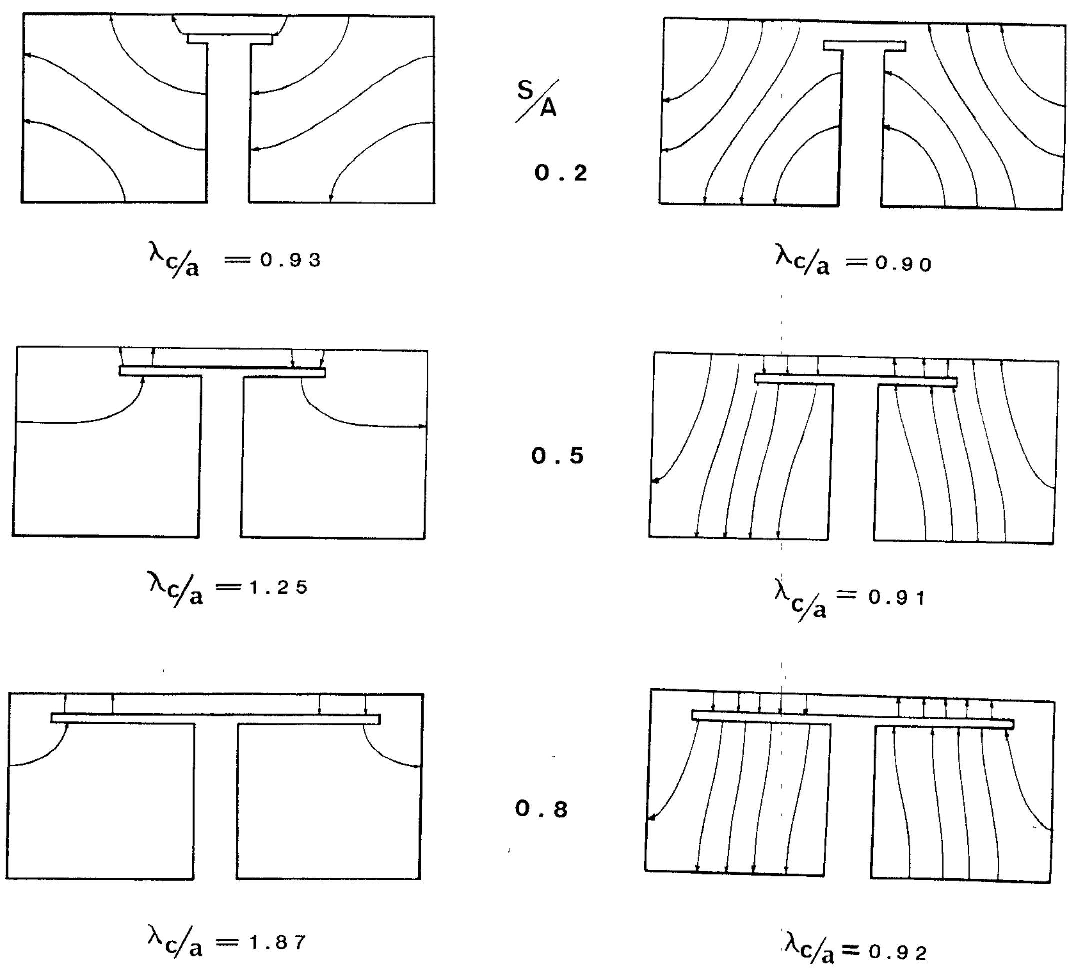

Fig. 6. Transverse electric field lines for the first two antisymmetric (electric wall) modes for the $t/b = 0.10$ case.

III. Results

Computer programs were written which implement (1) and (2) separately for the magnetic and electric wall cases. The programs compute the determinant of the matrix as a function of the unknown transverse wavenumber. The eigenvalue is determined by noting a change of sign between successive determinant values and then using quadratic interpolation. This scheme is simple to implement, and very accurate results can be obtained when the frequency axis is sampled sufficiently densely. Although this dense sampling requirement means that the method is computationally intensive, it also means that eigenvalues which are nearly degenerate are more likely to be found.

Figs. 3 and 4 illustrate the results we obtain when we apply our analysis to the geometry illustrated in the insets to the figures. (This geometry is the only case which was analyzed by both previous groups and therefore provides a meaningful case for comparisons between them.) Fig. 3 is a direct comparison between our results and those of Zhang and Joines, plotted on the scale used in [7]. The disagreement is quite large and is increasing as the value of the parameter (s/a) increases. Fig. 4 is a direct comparison between our results and those of Mazumder and Saha, plotted on the scale used in [6]. Although small discrepancies remain, it is clear that our results are in agreement with [6] and with the new results reported in [10].

In order to understand the source of the discrepancy between the references we tried plotting several other cases. Fig. 5 is our most successful attempt. The solid lines again indicate the bandwidth as reported in [7] and the symbols indicate the bandwidth defined by the difference in eigenvalues of the first two solutions associated with a magnetic wall in what we call region 1. These two modes are the first and third propagating modes of the structure.

Fig. 6 illustrates the transverse electric field lines for several values of (s/a) for the first two electric wall modes for one particular case $(t/b = 0.10)$ for the geometry investigated above. It is clear that in the limit as $s/a \rightarrow 0.0$ the waveguide becomes rectangular and these two modes reduce to the standard TE_{01} and TE_{20} modes. (In this limit the width of the base of the "T" must also go to zero.) As the parameter (s/a) increases, perturbation theory tells us that the modes become mixed and these simple labels are no longer meaningful. Although some resemblance to the TE_{20} field distribution appears in both cases, it is clear that using such labels is incorrect and may be misleading. It is, of course, equally incorrect to label the dominant mode as the TE_{10} and we have refrained from so doing. We prefer simply to label the modes in ascending order of cutoff frequency, i.e. TE_1, $TE_2, \cdots$, etc. However, we recognize that other schemes, such as that proposed in [9], may be more desirable in certain situations.

IV. Conclusions

The single T-septum waveguide has been analyzed using the Rayleigh–Ritz–Galerkin technique. The method is a variation of that used by two previous groups who reported conflicting results. The results of the new analysis agree closely with those of Mazumder and Saha [5], [6] and those of German and Riggs [10] and disagree with those of Zhang and Joines [7], [8]. The history of T-septum waveguide has been discussed.

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Optimized Method for Obtaining Permittivity and Conductivity Profiles of Microwave Materials

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***Abstract*—A new iterative method for obtaining the distribution of the relative permittivity $\epsilon_r(z)$ or electrical conductivity $\sigma(z)$ of microwave semiconducting materials is presented. The semiconducting material is fitted in a rectangular waveguide which is terminated by a variable short circuit. The reflection coefficients of the system are measured at a single frequency and at different positions of the moving short. The measured coefficients are used in the iterative process of solving the inverse problem by obtaining the functional gradient [1], [2]. The method takes into account continuous and discontinuous profiles.**

I. Introduction

Previously the solutions of the ill-posed electromagnetic inverse problems were discussed by Morozov [3] and Tikhonov [4]. Electromagnetic probing of an inhomogeneous stratified medium using time- and spectral-domain analyses of reflection coefficients is presented by Bolomey *et al.* [5]. The computed results are in general oscillating. The optimization technique is used with the direct method and the minimum of the cost function is reached after 65 iterations, which is large. The main drawbacks of the time- and spectral-domain methods are with discontinuous profiles. Approximate construction of the dielectric constant pro-

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